5 Decimal numbers

1 Decimal numbers

Decimal numbers such as 3.762 are used in situations in which we look for more precision than whole numbers provide.

As with whole numbers, a digit in a decimal number has a value which depends on the place of the digit. The places to the left of the decimal point are ones, tens, hundreds, and so on, just as with whole numbers. This table shows the decimal place value for various positions:

Note that adding extra zeros to the right of the last decimal digit does not change the value of the decimal number.

Place (underlined)	Name of Position
<u>1</u>.234567	Ones (units) position
1.<u>2</u>34567	Tenths
1.2<u>3</u>4567	Hundredths
1.23<u>4</u>567	Thousandths
1.234<u>5</u>67	Ten thousandths
1.2345<u>6</u>7	Hundred Thousandths
1.23456<u>7</u>	Millionths

Example:

In the number 3.762, the 3 is in the units place, the 7 is in the tenths place, the 6 is in the hundredths place, and the 2 is in the thousandths place.

- 3 is called the whole number portion
- 762 is the decimal portion

2 How to read decimal numbers.

We have to read the whole number, then the word "point" and the decimal numbers one by one.

Example:

In the number 2.34 we say two point three four

We read the number 0.057 as nought point zero five seven.

Exercise 1

Write in words the following:

21.456

0.77

0.0089

5.7254

3 Adding and subtracting decimals

To add or subtract decimals, line up the decimal points and then follow the rules for adding or subtracting whole numbers, placing the decimal point in the same column as above.

When one number has more decimal places than another, use 0's to give them the same number of decimal places.

Example:

Add 43.67 + 2.3

1) Line up the decimal points and adds a 0 on the right of the second:

2) Then add.

> 43.67
> 2.30
> 45.97

Example:

Subtract. 57.8 – 8.06

1) Line up the decimal points.

2) Add extra 0's

3) Subtract.

> 57.80
> 8.06
> 49.74

4 Multiplying decimal numbers

Multiplying decimals is just like multiplying whole numbers. The only extra step is to decide how many digits to leave to the right of the decimal point. To do that, add the numbers of digits to the right of the decimal point in both factors.

Example:

Multiply 23.56 x 34.1

$$
\begin{array}{r}
23.56 \\
34.1 \\
\hline
2356 \\
9424 \\
7068 \\
\hline
803.396
\end{array}
$$

Exercise 2 Calculate

a) 5.6 x 6.9

b) 12.37 x 76.78

c) –4.66 x 4.7

d) 0.345 (32.4 – 4.67)

5 Dividing whole numbers, with decimals

Continue the whole division adding zeros to the right of the number being divided until you get the amount of decimal digits required.

Example:

Divide 235:6 until the hundredth

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235   | 6
  55    39.16
   10
    40
     4
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Exercise 3 Calculate with two decimal digits

a) 56 : 7   b) 7634 : 34

c) – 679 : 32   d) 9783 : 127

6 Dividing decimals by decimals

To divide by a decimal, multiply that decimal by a power of 10 great enough
to obtain a whole number. Multiply the dividend by that same power of 10.
Then the problem becomes one involving division by a whole number instead
of division by a decimal.

Exercise 4 Calculate with three decimal digits

a) 56.7 : 2.34　　　　　　　b) 1432.3 : 0.42

c) – 12.34 : 3.5　　　　　　d) 1 : 1.2

7 Rounding Decimal Numbers

To round a number to any decimal place value, we want to find the number with zeros in all of the lower places that is closest in value to the original number. As with whole numbers, we look at the digit to the right of the place we wish to round to.

Note: When the digit 5, 6, 7, 8, or 9 appears in the ones place, round up; when the digit 0, 1, 2, 3, or 4 appears in the ones place, round down.

Exercise 5 Round:

1.17 to the nearest tenth

2.375 to the nearest hundredth

0.7084 to the nearest thousandth

12.87 to the nearest unit

151.504 to the nearest hundred

7478 to the nearest thousand

8 Writing a fraction as a decimal

Method 1 - Convert to an equivalent fraction whose denominator is a power of 10, such as 10, 100, 1000, 10000, and so on, then write in decimal form.

Examples:

1/4 = (1 × 25)/(4 × 25) = 25/100 = 0.25

3/20 = (3 × 5)/(20 × 5) = 15/100 = 0.15

Method 2 - Divide the numerator by the denominator. Round to the decimal place asked for, if necessary.

Examples:

13/4 = 13 ÷ 4 = 3.25

Convert 3/7 to a decimal number. Round it to the nearest thousandth.

We divide one decimal place past the place we need to round to, then round the result.

3/7 = 3 ÷ 7 = 0.4285…

That equals 0.429 when rounded to the nearest thousandth.

Exercise 6 Convert to a decimal. Round it to the nearest hundredth.

a) $\dfrac{4}{3}$ b) $\dfrac{13}{6}$

c) $\dfrac{11}{7}$ d) $\dfrac{7}{4}$

9 Repeating decimals

Every fraction can be written as a decimal.

For example, 1/3 is 1 divided by 3.

If you use a calculator to find 1 ÷ 3, the calculator returns 0.333333... This is called a **repeating decimal**. To represent the idea that the 3's repeat forever, one uses an arc

$$\frac{1}{3} = 0.33333.... = 0.\overset{\frown}{3}$$

In Britain they use a horizontal bar $\dfrac{1}{3} = 0.33333.... = 0.\overline{3}$

0 is the whole number portion

3333… is the decimal portion

3 is called the period or the recurring number, there is a period of one digit

Another example

$$\frac{13}{11} = 1.181818.... = 1.\overset{\frown}{18} \text{ or } \frac{13}{11} = 1.181818.... = 1.\overline{18}$$

1 is the whole number portion

1818… is the portion

18 is the period, this period has two digits

If there is no mark over the number it means that it has been an exact division. These numbers are called **regular numbers.**

Example 2/5 = 0.4 is a regular number

There can be decimals numbers without repeating decimals

Examples: 1.01001000100001……

$\pi = 3.14159........$

These are called irrational numbers and can not be written as fractions.

Exercise 7 Write as a decimal:

a) $\dfrac{17}{6}$ b) $\dfrac{13}{7}$

c) $\dfrac{131}{11}$ d) $\dfrac{71}{9}$

Exercises

1 Ellen earns £137.40 per week and after 4 weeks she gets an extra payment of £24.75, she spends £354.60 in this period. How much has she saved?

2 A student has been studying a total time of 4 h 35 min and during this time has been writing for 100 min. How long, in hours has he been studying without writing?

3 Susan cooked a cake and used 1.35kg of flour, 0.37kg of sugar, 3 eggs that weigh 80g each and 240g of milk. Which is the weight of the mixture?

4 I buy 7 mugs and pay 53.55 €. How much does each mug cost?

5 Henry had 83.40 €. He bought four tickets for the cinema at 6.50 each and 2 bags of pop corn at 2.25 each. How much money has he got left?

6 A breeder gives to each pig 0.65 kg of food for every 4 kg of body weight. There are 4 pigs of 75.8 kg, 56.4 kg, 75.4 kg and 89.3 kg. How much food must be prepared in total?

7 In a restaurant 7 friends are having a meal, the bill is £173.6 and each person contributes £25.50. What tip does the waiter receive?

8 A company produces items that are sold for £13.63 each, the daily production is 1275 items and the cost of production is £11,324.50. What is the daily income for the company?

9 Three partners share a benefit of £18538, the first one receives a third minus £173, the second one £5938.55. How much does the last one receive?

Solutions

Exercise 1

21.456 twenty one point four, five, six; 0.77 nought/zero point seven, seven; 0.0089 nought/zero point zero, zero, eight, nine; 5.7254 five point seven, two, five, four.

Exercise 2 a) 38.64, b) 949.7686, c) −21.902, d) 9.56685

Exercise 3 a) 8, b) 224.53, c) − 21.22, d) 77.03

Exercise 4 a) 24.231, b) 3410.238, c) − 3.526, d) 0.833

Exercise 5 1.2; 2.38; 0.708; 13; 200; 7000.

Exercise 6 a) 1.33, b) 2.17, c) 1.57, d) 1.75.

Exercise 7 a) $2.8\widehat{3}$, b) $1.\overline{857142}$, c) $11.9\widehat{0}$, d) $7.\widehat{8}$

Exercises

1 Ellen saves £219.75.

2 He has been studying without writing $2.913\widehat{6}$ h.

3 The weight of the mixture is 2.2 kg.

4 Each mug costs 7.65 €.

5 Henry has 52.90 € left.

6 He have to prepare 48.25 Kg of food.

7 The waiter receives a tip of £4.90.

8 The daily income for the company is £6053.75.

9 He receives 6593.12 £.